

An unitarized model for tetraquarks with a color flip-flip potential

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In this work, a color structure dependent flip-flop potential is developed for the two quarks and two antiquarks system. Then, this potential is applied to a microscopic quark model which, by integrating the internal degrees of freedom, is transformed into a model of mesons with non-local interactions. With this, the T matrix for the system is constructed and meson-meson scattering is studied. Tetraquarks states, interpreted as poles of the T matrix, both bound states and resonances, are found. Special emphasis is given to the truly exotic $qq\bar{Q}\bar{Q}$ system, but some results for the crypto-exotic $qQ\bar{q}\bar{Q}$ are also presented.

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1. Introduction

The existence of composite particles constituted by two quarks and two antiquarks, tetraquarks, is still debated. Although several experimental candidates [1, 2] have been advanced no one has been firmly established. From the theoretical point of view, these systems were studied mainly as a bound state of two quarks and two antiquarks [3, 4].

In this work we start with a microscopic model of two quarks and two antiquarks interacting through a four-body potential. By integrating the confined degrees of freedom we obtain a multi-channel model of mesons. This model is then used to find bound states and to construct the scattering T matrix, from where resonances are found.

2. Method

2.1. Microscopic potential

The static potential has been found on the lattice [5, 6]. It is given by a triple flip-flop potential, where its values corresponds to the confining

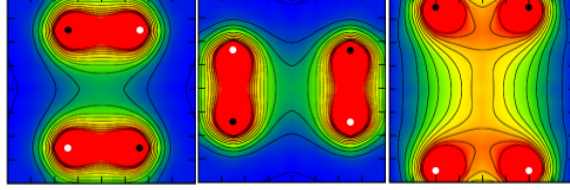


Fig. 1. The three possible string configurations for the ground state of a system of two static quarks and two static antiquarks

string disposition that minimizes the potential for a given configuration (see Fig. 1):

$$V_{FF} = \min(V_I, V_{II}, V_T) \quad (1)$$

V_I and V_{II} are the two-meson potentials

$$V_I = V_M(r_{13}) + V_M(r_{24}) \quad (2)$$

$$V_{II} = V_M(r_{14}) + V_M(r_{23}) \quad (3)$$

where V_M is the quark-antiquark potential in a meson, which is well described by the Cornell potential $V_M = K - \frac{\gamma}{r} + \sigma r$.

V_T is the tetraquark potential, given by

$$V_T = 2K - \gamma \sum_{i < j} \frac{C_{ij}}{r_{ij}} + \sigma L_{min}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$$

where $C_{ij} = 1/2$ between two quarks or two antiquarks and $C_{ij} = 1/4$ between a quark and an antiquark. L_{min} is the minimal length of the string linking the four particles.

Two linearly independent color singlets can be formed from two quarks and two antiquarks, say the two meson-meson states: $|\mathcal{C}_I\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_i \bar{Q}_j\rangle$ and $|\mathcal{C}_{II}\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_j \bar{Q}_i\rangle$, or the color anti-symmetric and symmetric states $|\mathcal{A}\rangle = \frac{\sqrt{3}}{2}(|\mathcal{C}_I\rangle - |\mathcal{C}_{II}\rangle)$ and $|\mathcal{S}\rangle = \sqrt{\frac{3}{8}}(|\mathcal{C}_I\rangle + |\mathcal{C}_{II}\rangle)$. We need a 2×2 matrix potential to be possible a transition between the two states. So, we have to know the first excited potential of the system, as well as the color structure of both states.

The color vector of the ground state could either be $|\mathcal{C}_I\rangle$ when $V_{FF} = V_I$, $|\mathcal{C}_{II}\rangle$ when $V_{FF} = V_{II}$ or $|\mathcal{A}\rangle$ when $V_{FF} = V_T$. As for the excited state, we know it has to be orthogonal to the ground one since the potential is hermitian. So we have $|\bar{\mathcal{C}}_I\rangle$ when $V_{FF} = V_I$, $|\bar{\mathcal{C}}_{II}\rangle$ when $V_{FF} = V_{II}$ and $|\mathcal{S}\rangle$ when $V_{FF} = V_T$, with $\langle \mathcal{C}_A | \bar{\mathcal{C}}_A \rangle = 0$. We assume that the value of the excited state is the second lowest of the three potentials. This way we obtain the potential of the system.

2.2. From Quarks to Mesons

Since we study meson-meson interaction, the natural choice for the color structure basis is the $|\mathcal{C}_I\rangle$ and $|\mathcal{C}_{II}\rangle$. Note that in this basis $g_{AB} \equiv \langle \mathcal{C}_A | \mathcal{C}_B \rangle \neq \delta_{AB}$

$$g = \begin{pmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{pmatrix}$$

Expanding the color states $\Psi = \Psi^A \mathcal{C}_A$, we arrive at the Schrödinger equation

$$g_{AB} \hat{T}_q \Psi^B + \hat{V}_{AB} \Psi^B = E g_{AB} \Psi^B \quad (4)$$

Since we want a theory of mesons, we must have the kinetic energy of both meson sectors, and not the kinetic energy of quarks $T_I = T_q + V_I \neq T_{II} = T_q + V_{II} \neq T_q$. For this, we define the kinetic energy of meson in a way that is both hermitian and gives the correct asymptotic states:

$$\hat{T}_S = \begin{pmatrix} \hat{T}_I & \frac{\hat{T}_I + \hat{T}_{II}}{6} \\ \frac{\hat{T}_I + \hat{T}_{II}}{6} & \hat{T}_{II} \end{pmatrix}$$

and

$$\hat{V}_S = \begin{pmatrix} V_{11} - V_I & V_{12} - \frac{V_I + V_{II}}{6} \\ V_{12} - \frac{V_I + V_{II}}{6} & V_{22} - V_{II} \end{pmatrix}$$

This gives a new Schrödinger equation with the same form. The components Ψ^A are then expanded in two meson states and so we obtain the equation

$$\hat{T}_{\alpha\beta} \psi^\beta + \hat{V}_{\alpha\beta} \psi^\beta = E g_{\alpha\beta} \psi^\beta \quad (5)$$

where the greek letter index includes the color index A and the remaining quantum numbers index i . The potential V has the form

$$\begin{aligned} \hat{V}_{AiAj} \psi^{Aj} &= V_{ij}(\mathbf{r}) \psi^{Aj}(\mathbf{r}) \\ \hat{V}_{AiBj} \psi^{Bj} &= \int d^3 \mathbf{r}'_B v_{ij}(\mathbf{r}_A, \mathbf{r}'_B) \psi^{Bj}(\mathbf{r}'_B) \quad \text{when } A \neq B \end{aligned}$$

. $T_{\alpha\beta}$ and $g_{\alpha\beta}$ have similar structures.

2.3. Asymptotic behavior

Writing, each component as $\psi^\alpha(r) = \frac{u^\alpha(r)}{r} Y_{l_\alpha m_\alpha}$, the asymptotic behavior of $u^\alpha(r)$ is

$$u^\alpha(r) \rightarrow A_{i\alpha} \sqrt{\frac{\mu_\alpha}{k_\alpha}} \sin(k_\alpha r - \frac{l_\alpha \pi}{2} + \varphi_{i\alpha}) + f_{i\alpha} e^{i(k_\alpha r - \frac{l_\alpha \pi}{2})} \quad (6)$$

This leads to the definition of the scattering T matrix for this system

$$T_{ij} = \sum_{\alpha} \sqrt{\frac{k_{\alpha}}{\mu_{\alpha}}} A_{i\alpha}^* e^{-i\varphi_{i\alpha}} f_{j\alpha} \quad (7)$$

To calculate it, we first generate N_{open} eigenfunctions of the \hat{T}_s operator $\hat{T}_s \Psi_0 = Eg \Psi_0$, where N_{open} is the number of open channels. Then the base is orthogonalized with the Gram-Schmidt procedure, using as inner product

$$\langle \Psi_{0i} | \Psi_{0j} \rangle = \sum_{\alpha} A_{i\alpha}^* A_{j\alpha} \cos(\varphi_{i\alpha} - \varphi_{j\alpha})$$

This product is a direct consequence of the asymptotic behavior Eq. 6. $A_{i\alpha}$ are computed by fitting the long range behavior of the generated functions.

We calculate the Ψ_i by solving Eq. 5 with $\Psi_i = \Psi_{0i} + \chi_i$:

$$(\hat{T} + \hat{V})\chi_i = Eg\chi_i - V\Psi_{0i}$$

From the long distance behavior of χ_i we find $f_{i\alpha}$ and calculate the T matrix with Eq. 7.

By continuing the definition of the T matrix into the complex energy plane we find it's poles which are tetraquark resonances.

2.4. Bound states

We need a very large box to be able to accurately find bound states, if we use Dirichlet boundary conditions and the bound states have a very small binding energy, having therefore a large spatial extension. To solve Eq. 5 using finite differences we employ boundary conditions that depend on the energy

$$[H + B(E)]u = Egu$$

and try to find a zero on the determinant of the matrix $H + B(E) - Eg$. Employing the Newton's method, it is found with the iteration

$$E^{(n+1)} = E^{(n)} - \frac{1}{\text{Tr}[(H + B(E) - Eg)^{-1}(B'(E) - g)]}$$

3. Results

In this work we neglect all spin and dynamical quark effects. The meson kinematics is non-relativistic.

$m_x(\text{GeV})$	$B(\text{MeV})$
1.30	$\simeq 0$
1.00	-0.95
0.70	-7.91
0.40	-48.54

Table 1. Binding energies of the $q\bar{q}b\bar{b}$ bound states for different quark masses

$m_x(\text{GeV})$	$E(\text{GeV})$	N_{open}	$m_x(\text{GeV})$	$E(\text{GeV})$	N_{open}
1.30	12.998 - 0.0179i	2	0.70	11.545 - 0.237i	1
1.00	12.505 - 0.0192i	2		12.019 - 0.033i	2
0.70	12.050 - 0.0215i	2	0.40	11.431 - 0.024i	1
0.40	11.666 - 0.0171i	2		11.687 - 0.114i	2

Table 2. Resonances for the $xx\bar{b}\bar{b}$ system(left) and for the $xb\bar{x}\bar{b}$ (right)

3.1. Exotic channels

For the exotic $qq\bar{Q}\bar{Q}$ system, we consider the wave-function to be of the type

$$\Psi = \Phi(\boldsymbol{\rho}_{13}, \boldsymbol{\rho}_{24})\psi(\mathbf{r}_{13,24})\mathcal{C}_I + \xi\Phi(\boldsymbol{\rho}_{14}, \boldsymbol{\rho}_{23})\psi(\mathbf{r}_{14,23})\mathcal{C}_{II}$$

, where $\xi = \pm 1$. This wavefunction includes space and color degrees of freedom, but not spin. The functions Φ must have a definite symmetry for the exchange of it's arguments: $\Phi(\mathbf{y}, \mathbf{x}) = s\Phi(\mathbf{x}, \mathbf{y})$ with $s = \pm 1$. This way, when we apply the exchange operators of color and space P_{ij}^{RC} we obtain

$$\begin{aligned} P_{12}^{RC}\Psi &= \xi(-1)^{L_r} s \Psi \\ P_{34}^{RC}\Psi &= \xi \Psi \end{aligned}$$

Including spin and since wave-function must be anti-symmetric for quark and antiquark exchanges, we have $P_{12}\Psi = (-1)^{1+S_{12}}\xi(-1)^{L_r} s \Psi = -\Psi$ and $P_{34}\Psi = (-1)^{1+S_{34}}\xi\Psi = -\Psi$. In this work, we choose $\xi = 1$, $s(-1)^{L_r} = 1$ and $L = 0$. This gives, $S_{12} = S_{34} = 0$ and so $S = 0$. Consequently, we have $J = 0$. We also choose states of positive parity, only.

With $m_{\bar{Q}} = m_b = 4.7 \text{ GeV}$, and varying the mass of the quark from $m_x = 0.40 \text{ GeV}$ to $m_x = 1.3 \text{ GeV}$, we find bound states for all the quark masses. Results for the binding energy are given on table 1 and the wave-functions of the ground state component are shown in Fig. 2. For this system we find resonances between the opening of the the second and third thresholds. Their complex energies are shown in Table 2.

Setting $m_{\bar{Q}} = 1.3 \text{ GeV}$ and similar quark masses, no bound states or resonances are found.

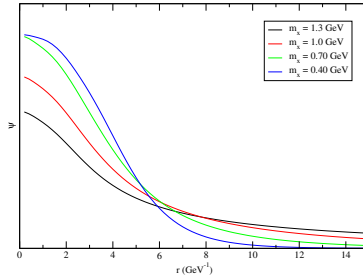


Fig. 2. Left: Bound state wavefunction for different masses of the lightest quark in the $xx\bar{b}\bar{b}$ system.

3.2. Crypto-exotic channels

We also study the crypto-exotic $qQ\bar{q}\bar{Q}$ system, for $m_Q = m_b = 4.7$ GeV and $m_q = m_x$ varying from 0.4 to 1.3 GeV. We don't find any bound states and only find resonances for $m_x = 0.40$ GeV and $m_x = 0.70$ GeV. Their energies are displayed in Table 2.

4. Conclusion

An unitarized method to compute the meson-meson scattering was developed. With it we were able to find bound states and resonances for the $0^+ xx\bar{b}\bar{b}$ system. For the $xb\bar{x}\bar{b}$ system, only resonances were found and for sufficiently small m_x . Refinements should be easy to include in this model.

Our results however, seems to disagree with lattice results, because the bound state for exotic system has $S_{12} = 0$ and so is a scalar isotriplet, but, according to [7] such a system should be repulsive. More work is needed to understand the source of this discrepancy and whether it is problem with the potential model or with the approach itself.

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